Analysis of Waveguide Post Configurations: Part 1—Gap Immittance Matrices

J. S. JOSHI AND J. A. F. CORNICK

Abstract—**The reaction concept in electromagnetics is used to analyze two useful kinds of post-mounting structure in rectangular waveguide. The first is the general multiple-post arrangement with a gap in each post, the second being a single post with multiple gaps. The analysis derives equations for calculating the elements of the gap immittance matrices for these two structures.**

Introduction

RECTANGULAR waveguide post-mounting structures have wide practical applications and are commonly used in transferred-electron device and IMPATT oscillators, often with an associated varactor tuning diode. In such arrangements it is usual to post mount the device in the guide, providing power or biasing via a suitable choke to each post. Many other uses can be visualized such as multiple source excitation, p-i-n diode attenuators, and passive filter elements.

As a consequence of the practical applications of such structures, a number of characterizations have been presented. Those considered so far have contained some limitations on the generality of the structure or in the description of the equivalent network. In-line configurations, where two posts are sited along the central z axis of the guide, have been analyzed by some authors [1], [2] to explain the performance of Gunn oscillators. Templin and Gunshor [1] used a modified Eisenhart and Khan [3] analysis for a single-post structure, while Dean and Howes [2] used Marcuvitz [4] results for the reactance of a post with the gap located at the extremity. The post reactances were calculated in isolation and were coupled by the dominant TE_{10} mode using the waveguide length impedance translation. Such techniques do not include the effect of evanescent mode coupling between the two posts. This omission may be justified when the posts are widely separated, but since small separations may be very desirable in practice, the method will be least useful in just these cases.

The other special arrangement which has been considered is the coplanar post structure in which two posts are located in one plane normal to the direction of propagation. Even in this case the structure considered has been of a symmetrical nature with respect to both the post and gap positions [5]. Chang and Khan [6] analyzed the case of coplanar location of two unequal flat strips. The usefulness of the analysis is restricted as finite gaps in the strips are not considered.

It is thus seen that all analyses to date have suffered from a loss of generality in the structure. Here an attempt is made to characterize the completely general multipost mounting arrangement in the form of a gap impedance matrix for this structure.

The alternative mounting arrangement, although not as practical, in which both devices are mounted in a single post has previously [7] been considered for the case of symmetrically placed gaps, while Eisenhart [8] has given an analysis and equivalent circuit of the two-gap single-post waveguide mount. This alternative arrangement is also considered here with the generality of multiple gaps in the post and in this case a gap admittance matrix derived.

Fig. 1 shows the location and parameters of a general post i. In the multipost case the post and gap variables are s_i, d_i, g_i, h_i, and L_i. In the single-post multigap case the variables are g_i and h_i since s, d, L will be fixed for the single post.

The Reaction Concept

The reciprocity theorem in electromagnetic systems under certain well-defined conditions equates the response at one source due to a second source to the response at the second source due to the first. Considering two sources $(\boldsymbol{J}_a,\boldsymbol{M}_a)$ and $(\boldsymbol{J}_b,\boldsymbol{M}_b)$ in a linear and isotropic medium enclosed in a volume V, the general statement of reciprocity becomes

$$\iiint (\boldsymbol{E}_a \cdot \boldsymbol{J}_b - \boldsymbol{H}_a \cdot \boldsymbol{M}_b)\, dV = \iiint (\boldsymbol{E}_b \cdot \boldsymbol{J}_a - \boldsymbol{H}_b \cdot \boldsymbol{M}_a)\, dV. \quad (1)$$

The volume integrals appearing on both sides of (1) have been given the name reaction [9] and are measures of the coupling between the two sources. Thus the reaction of field a on source b is given by the scalar

$$\langle a,b \rangle = \iiint (\boldsymbol{E}_a \cdot \boldsymbol{J}_b - \boldsymbol{H}_a \cdot \boldsymbol{M}_b)\, dV. \quad (2)$$

The reciprocity relationship can then be written

$$\langle a,b \rangle = \langle b,a \rangle.$$

The self-reaction $\langle a,a \rangle$ is the reaction of field a on source a.

Applications to Circuits

Ideal sources in circuit theory are independent of the load, and their circuit reactions can be written [10] as

$$\langle a,b \rangle = \begin{cases} -V_a{}^b I_b, & \text{where } b \text{ denotes current source} \\ -V_b I_a{}^b, & \text{where } b \text{ denotes voltage source.} \end{cases} \quad (3)$$

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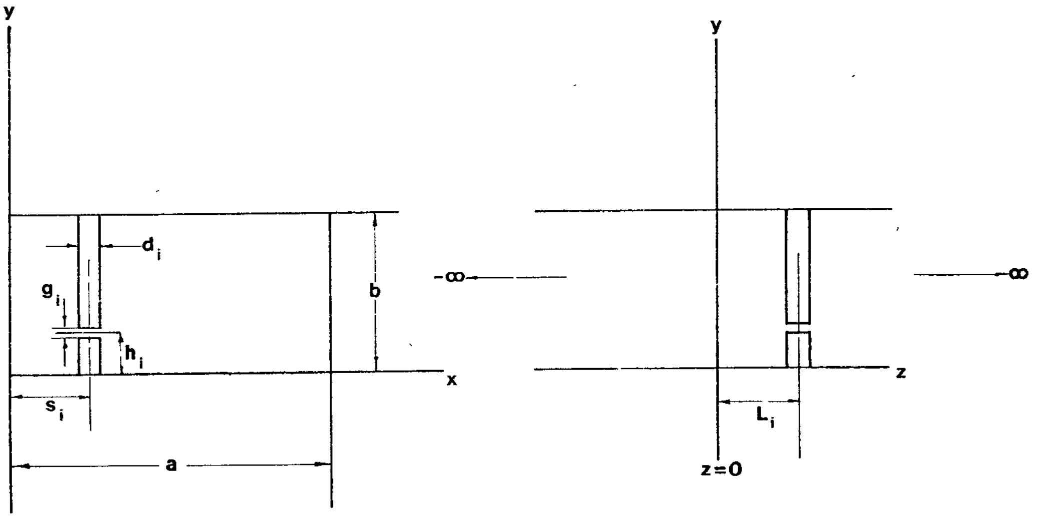

Fig. 1. Post i location and parameters in rectangular infinite guide.

Thus for the unit current source $I_b = 1$, $\langle a,b \rangle$ is a measure of V_a^b, the voltage at b due to source a. Similarly, for unit voltage source $V_b = 1$, $\langle a,b \rangle$ is a measure of I_a^b which is the current at b due to source a.

The equations for a linear N-port in terms of open-circuit impedances Z_{ij} are

$$V_i = \sum_{j=1}^{N} Z_{ij} I_j, \qquad i = 1 \text{ to } N$$

where V,I are the excitations at the ports and the N-port is completely characterized by the impedance matrix $[Z_{ij}]$.

The self-impedance term Z_{ii} for port i is the ratio of the voltage and current at port i with all other ports open circuited. The mutual impedance term Z_{ij} $(i \neq j)$ is the ratio of the voltage at port i to current at port j with all ports $(\neq j)$ open circuited.

The general impedance element for the current source excitation is

$$Z_{ij} = \frac{V_{ij}}{I_j}$$

where V_{ij} is the open-circuit voltage at port i and I_j is the current at port j with all ports $(\neq j)$ open. Using (3),

$$Z_{ij} = -\frac{\langle j,i \rangle}{I_i I_j}. \qquad (4)$$

The reciprocity theorem implies that $Z_{ij} = Z_{ji}$. The elements of the impedance matrix $[Z_{ij}]$ are thus the various reactions between the (i,j) pairs of current sources at the network ports.

Applications to Antennas

For a perfectly conducting antenna excited by a current source I_a, the current distribution in the antenna is such that the tangential component of the total electric field vanishes on the surface of the antenna. If a trial current density distribution $\boldsymbol{J}_a^t$ is assumed in the antenna, the input impedance will be given by (2) and (4) [10]

$$Z_{in} = Z_{aa} = -\frac{\langle a,a \rangle}{I_a^2} = -\frac{1}{I_a^2}\int_S \boldsymbol{E}_a \cdot \boldsymbol{J}_a^t \, dS. \qquad (5)$$

For the case of two antennas a,b with currents I_a and I_b, respectively, the mutual impedance is given by (2) and (4)

$$Z_{ba} = -\frac{\langle a,b \rangle}{I_a I_b} = -\frac{1}{I_a I_b}\int_S \boldsymbol{E}_a \cdot \boldsymbol{J}_b^t \, dS \qquad (6)$$

where $\boldsymbol{E}_a$ is the field at b due to current density $\boldsymbol{J}_a^t$ in antenna a and $\boldsymbol{J}_b^t$ is the trial current density in antenna b. It can be shown that the impedances are stationary with respect to variations in the trial current density distributions [9], [10].

Multipost Structure in Rectangular Waveguide

Following Eisenhart and Khan [3] the post i is replaced by an equivalent strip of width 1.8 times the diameter. Again the assumptions of uniform current density distribution across the strip and uniform electric field distribution in the gap are made [3].

The notation in the text is identical to that used by Eisenhart and Khan [3]. Their method is used to evaluate the terminal current of the antenna (gap port). The reaction concept is then used to derive the general expression for the impedance matrix elements.

The Dyadic Green's Function: Since the posts are y directed, it is only necessary to consider y-directed current distributions, and the form of this function for an infinite rectangular waveguide becomes

$$\bar{G}(r/r') = yy \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \cdot \frac{(2-\delta_n)(k^2 - k_y^2)\exp(-\Gamma_{mn}|z-z'|)}{abk^2\Gamma_{mn}} \cdot \sin k_x x \cos k_y y \sin k_x x' \cos k_y y'$$

where

$$k_x = \frac{m\pi}{a} \quad k_y = \frac{n\pi}{b} \quad k = \frac{2\pi}{\lambda}$$

$$\Gamma_{mn} = (k_x^2 + k_y^2 - k^2)^{1/2}$$

$$\delta_n = 1, \quad \text{for } n = 0$$
$$= 0, \quad \text{for } n \neq 0.$$

Trial Current Density Distribution: The problem of choosing a suitable trial current density can be made simpler by writing the current density in the form of a general orthogonal set of circular functions corresponding to Green's function; thus for post i,

$$J_i^t(r) = yJ_{0i}u_i(x)u_i(y)\delta(z - L_i)$$

where

$$u_i(x) = \sum_{f=1}^{\infty}\left(\frac{2-\delta_f}{a}\right)\left(A_{if}^x \cos\frac{f\pi x}{a} + B_{if}^x \sin\frac{f\pi x}{a}\right)$$

$$u_i(y) = \sum_{I=0}^{\infty}\left(\frac{2-\delta_I}{b}\right)\left(A_{iI}^y \cos\frac{I\pi y}{b} + B_{iI}^y \sin\frac{I\pi y}{b}\right).$$

The trial current density distribution selection now reduces to finding values of the normalized expansion coefficients. The y-distribution coefficients can be left in general form, but if the posts are replaced by equivalent flat strips of widths $w_i = 1.8d_i$, then the assumption of uniform current density across the strip width gives specific values to coefficients A_{if}^x and B_{if}^x as

$$A_{if}^x = w_i \cos\frac{f\pi s_i}{a}\frac{\sin\theta_{if}}{\theta_{if}}$$

$$B_{if}^x = w_i \sin\frac{f\pi s_i}{a}\frac{\sin\theta_{if}}{\theta_{if}}$$

where

$$\theta_{if} = \frac{f\pi w_i}{2a}.$$

This assumption places an upper limit on the ratio of post diameter to waveguide broad dimension consistent with acceptable errors. It is not likely to be a problem in practical cases due to the conveniently available sizes of packages for semiconductor devices. It is expected that for strip widths $w_i < a/4$ the assumption is justified.

Electric Field Distribution: The electric field is given by

$$E(r) = -j\omega\mu \int_v \bar{G}(r/r')\cdot J(r')\,dV'$$

which results in

$$E_i(r) = -y\frac{j\eta J_{0i}}{abk}\sum_{n=0}^{\infty}\sum_{m=1}^{\infty}\frac{(2-\delta_n)(k^2-k_y^2)}{\Gamma_{mn}} \cdot A_{in}^y B_{im}^x \sin k_x x \cos k_y y \exp(-\Gamma_{mn}|z - L_i|).$$

Gap Electric Field: The assumption is made that a constant spatial electric field exists in each gap. In most practical cases the gap sizes in relation to the waveguide height are small, and this will not lead to excessive error. For gap size $g_i < b/4$, it is expected that the assumption will be justified. Defining

$$E_{gi} = -y\frac{V_i}{g_i}v_i(y)v_i(x)\delta(z - L_i)$$

where

$$v_i(x) = 1, \quad \text{for } s_i - \frac{w_i}{2} \le x \le s_i + \frac{w_i}{2}; \quad z = L_i$$

$$= 0, \quad \text{elsewhere.}$$

The y distribution is

$$v_i(y) = \sum_{p=0}^{\infty}\left(\frac{2-\delta_p}{b}\right)S_{ip}\cos\frac{p\pi y}{b}.$$

The normalized expansion coefficients S_{ip} are given by

$$S_{ip} = g_i \cos\frac{p\pi h_i}{b}\frac{\sin\phi_{ip}}{\phi_{ip}}$$

where

$$\phi_{ip} = \frac{p\pi g_i}{2b}.$$

The input currents at the antenna terminals can be obtained by calculating the incident power on each of the gaps, respectively, for each n. For gap i at $z = L_i$,

$$\int_s E_{gi}(r)\cdot J_i(r)\,dS = -V_i J_{0i}A_{in}^y\left(\frac{2-\delta_n}{b}\right)\cos\frac{n\pi h_i}{b}\frac{\sin\phi_{in}}{\phi_{in}}w_i$$

or

$$I_{in} = \left(\frac{2-\delta_n}{b}\right)A_{in}^y J_{0i}w_i \cos\frac{n\pi h_i}{b}\frac{\sin\phi_{in}}{\phi_{in}}.$$

The Impedance Matrix Elements: Having determined the currents at the antenna terminals (gap ports), the self-impedance of gap port i can be calculated for each n using (5). Thus

$$Z_{iin} = -\frac{1}{I_{in}^2}\int_{x=0}^{a}\int_{y=0}^{b} E_i(r)\cdot J_i^t(r)\,dx\,dy, \qquad \text{at } z = L_i$$

$$= j\eta\frac{b}{a}\frac{(k^2-k_y^2)}{k(2-\delta_n)}\sum_{m=1}^{\infty}\frac{(k_{ipm}/k_{ign})^2}{\Gamma_{mn}}$$

where

$$k_{ipm} = \sin k_x s_i \frac{\sin\theta_{im}}{\theta_{im}}$$

$$k_{ign} = \cos k_y h_i \frac{\sin\phi_{in}}{\phi_{in}}.$$

The mutual impedance term for each n is given by [see (6)]

$$Z_{jin} = -\frac{1}{I_{in}I_{jn}}\int_{x=0}^{a}\int_{y=0}^{b} E_i(r)\cdot J_j^t(r)\,dx\,dy, \qquad \text{at } z = L_j$$

$$= j\eta\frac{b}{a}\frac{(k^2-k_y^2)}{k(2-\delta_n)}\sum_{m=1}^{\infty}\frac{k_{ipm}k_{jpm}}{k_{ign}k_{jgn}}\frac{\exp(-\Gamma_{mn}L_{ij})}{\Gamma_{mn}}$$

where L_{ij} is the distance between the i,j post planes measured along the z axis. Writing the expression for Z_{ij} at $z = L_i$ confirms that $Z_{ijn} = Z_{jin}$. The general expression for the elements of the N-port impedance matrix for the nth spatial harmonic can be written as

$$Z_{ijn} = \sum_{m=1}^{\infty} Z_{mn}\left(\frac{k_{ipm}k_{jpm}}{k_{ign}k_{jgn}}\right)\exp(-\Gamma_{mn}L_{ij}) \tag{7}$$

where Z_{mn} is the mode pair impedance term given by

$$Z_{mn} = j\eta\frac{b}{a}\frac{k^2-k_y^2}{k(2-\delta_n)}\frac{1}{\Gamma_{mn}}.$$

The Coupling Network for the nth Spatial Harmonic: The impedance matrix elements given in (7) can be used to obtain the elements of the gap multiport network for each n. As expected, the self-impedance terms Z_{ii} represent the driving point impedances at gaps i in the absence of all other posts. The expressions for these impedances therefore do not contain any terms in L ($L_{ii} = 0$). The expression for the mutual terms Z_{ij} ($i \neq j$), however, takes into account the z separation of the posts and contains the $\exp(-\Gamma_{mn}L_{ij})$ term in addition to the post and gap coupling factors of the two posts.

For a coplanar post pair where $L_{ij} = 0$, the mutual impedance term can represent strong coupling between the two-gap ports. For nonzero L_{ij} values the propagating mode terms will be dominant, corresponding to $\Gamma_{m'n'}L_{ij} = j\beta_{m'n'}L_{ij}$, while the evanescent mode terms will be attenuated by the factor $\exp(-\Gamma_{mn}L_{ij})$. This factor tends to zero as L_{ij} increases with the consequent elimination of the coupling due to these modes.

The terms of the impedance matrix elements Z_{ijn} for the nth spatial harmonic can be used to represent the coupling to the various (m,n) mode pair impedance ports from $m = 1$–∞ through the post and gap coupling factors and the exponential distance factors. The matrix elements themselves can be used to construct the coupling network between the N gap ports for the nth spatial harmonic.

The Complete Network

Equation (7) defines the elements to form the coupling networks between the gap ports for all $n = 0$–∞. The network for such n establishes the relationship between the gap voltages and the nth spatial harmonic components of current. The total current at a gap port is the sum of all individual currents for $n = 0$–∞. The complete network is thus the parallel combination of all the individual networks.

In effect, the complete network is a multiport network consisting of the N gap ports and the infinite number of (m,n) mode pair impedance ports. The accessible ports are the gap ports and the propagating mode pair ports at the waveguide arms.

The General Single-Post Multigap Structure

In this case it is more appropriate for a general gap i and series connection of the antennas to consider a matrix representation in the form of short-circuit admittance rather than open-circuit impedance. In terms of the reaction concept, the elements of this admittance matrix can be expressed as circuit reactions between the unit voltage sources at the gaps. The electromagnetic field formulation of this problem will then be in terms of magnetic current source excitations (equivalent to a voltage source in circuit theory) at the gap locations and the dyadic Green's function corresponding to the magnetic current source.

However, the admittance expressions for the network can be obtained without resort to this approach. The network equations of a linear N-port can be written in terms of short-circuit admittance as

$$I_i = \sum_{j=1}^{N} Y_{ij}V_j, \qquad i = 1 \text{ to } N$$

where V,I are the excitations at the gap ports. The self-admittance term Y_{ii} for port i is the ratio of current to voltage at port i with all other ports short circuited, while the transfer admittance Y_{ij} is the ratio of short-circuit current in port i to the voltage at port j with all ports ($\neq j$) short circuited. Also, the transfer impedance Z_{ij}' is defined as the ratio of voltage at port i to the current at port j when short circuited. Z_{ji}' is defined similarly.

Thus $Y_{ij} = 1/Z_{ji}'$ and $Y_{ji} = 1/Z_{ij}'$. When these definitions are applied to the post configuration, the self-admittance Y_{ii} of port i reduces to the admittance at gap i when the post contains only that gap. This is equivalent to Eisenhart and Khan's [3] expression for the single-post one-gap case.

The transfer admittance Y_{ij} can be expressed in terms of the transfer impedance Z_{ji}' which for the post structure can, in turn, be expressed in terms of the reaction $\langle i,j \rangle$. Thus

$$Y_{ij} = \frac{-1}{\langle i,j \rangle / I_i I_j} = \frac{-I_i I_j}{\int_s \boldsymbol{E} \cdot \boldsymbol{J}^t \, dS} \tag{8}$$

where $\boldsymbol{E}$ is the electric field at the post due to the trial current $\boldsymbol{J}^t$ in the antenna. The reciprocity relation gives $Y_{ij} = Y_{ji}$.

The Admittance Expressions: Using a similar analysis to that of the multiple-post structure together with (8), the admittance can be written as

$$Y_{ijn} = \left[\sum_{m=1}^{\infty} Z_{mn} \frac{k_{pm}^{\;2}}{k_{ign}k_{jgn}} \right]^{-1}. \tag{9}$$

In this case there is only one post, so that there is a single value k_{pm} of the post coupling factor.

Network Representation for the nth Spatial Harmonic: The admittance matrix elements given by (9) can be used to construct the coupling network between the gap ports for each n. The elements of such a network represent coupling to all the waveguide mode pair impedance ports from $m = 1$ to ∞ for a fixed value of n. The coupling is dependent on the post coupling factor and the gap coupling factors. The matrix elements can be used to construct the coupling network between the gap ports for the nth spatial harmonic.

The complete network is composed of all the networks for $n = 0$–∞ connected in parallel. The accessible ports are the N gap ports and the propagating mode pair ports at the waveguide arms.

Conclusions

The reaction concept has been used to obtain network representation of two general waveguide post structures, one consisting of N posts with one gap in each and the other a single post with N gaps. The network representation is valid throughout the frequency range where multimode propagation in the waveguide may occur, and can be applied to the cases where the immittances at the gap terminals are of interest. Examples are multiply excited or tunable solid-state sources. The generality of the concept could enable the method to be used as an alternative approach to the analysis of other structures.

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Analysis of Waveguide Post Configurations: Part II—Dual-Gap Cases

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***Abstract*—The analysis given in Part I [1] is applied to two particular structures of general interest. The first is the two-post case, each post having one gap, and the second is the single-post with two gaps. The impedance or admittance matrix elements are used to construct the gap port networks which are then used to obtain the waveguide obstacle representations. These latter representations are used to obtain experimental confirmation of the analysis.**

INTRODUCTION

THE general two-post and single-post two-gap structures in infinite rectangular waveguide are of special significance in the design of microwave solid-state sources. The gap immittance matrices derived in [1] are applied here to these specific cases. First, the nature of the coupling network between the two-gap ports is discussed, and these coupling networks are then used to derive the impedance presented by the post structure to the dominant H_{10} mode in the guide. Results of experimental work carried out on some general post arrangements are also included in support of the theory. The relevant matrix expressions derived in [1] are given here for convenient reference with all the quantities defined.

For the structure of Fig. 1(a),

$$Z_{ijn} = \sum_{m=1}^{\infty} Z_{mn} \left(\frac{k_{ipm} k_{jpm}}{k_{ign} k_{jgn}} \right) \exp\left(-\Gamma_{mn} L_{ij}\right) \tag{1}$$

and for the structure of Fig. 1(b),

$$Y_{ijn} = \left[\sum_{m=1}^{\infty} Z_{mn} \left(\frac{k_{pm}{}^2}{k_{ign} k_{jgn}} \right) \right]^{-1} \tag{2}$$

where

$$Z_{mn} = j\eta \frac{b}{a} \frac{(k^2 - k_y{}^2)}{(2 - \delta_n)k} \frac{1}{\Gamma_{mn}}$$

$$k_{ipm} = \sin k_x s_i \frac{\sin \theta_{im}}{\theta_{im}} \qquad k_{ign} = \cos k_y h_i \frac{\sin \phi_{in}}{\phi_{in}}$$

$$\theta_{im} = \frac{m\pi w_i}{2a} \qquad \phi_{in} = \frac{n\pi g_i}{2b} \qquad w_i = 1.8 d_i$$

$$k_x = \frac{m\pi}{a} \qquad k_y = \frac{n\pi}{b} \qquad k = \frac{2\pi}{\lambda}$$

$$\Gamma_{mn} = (k_x{}^2 + k_y{}^2 - k^2)^{1/2}$$

$$\delta_n = 1, \quad \text{for } n = 0$$
$$= 0, \quad \text{for } n \neq 0, \qquad i,j = 1,2.$$

THE TWO-POST STRUCTURE

The elements of the impedance matrix for an nth spatial harmonic mode given by (1) can be used to construct the coupling network between the two-gap ports. This network is preferred in T form since impedance elements are considered. All the various mode pair impedance ports for $m = 1$–∞ for the given value of n are coupled to the two-gap ports through the post and gap coupling factors and the exponential distance factors.

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